

Absolute Distance Measurements Using the Optical Comb of a Femtosecond Pulse Laser

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We describe a new way of implementing absolute displacement measurements by exploiting the optical comb of a femtosecond pulse laser as a wavelength ruler. The optical comb is stabilized by locking both the repetition rate and the carrier offset frequency to an Rb clock of frequency standard. Multiwavelength interferometry is then performed using the quasi-monochromatic beams of well-defined generated wavelengths by tuning an external cavity laser diode consecutively to preselected light modes of the optical comb. This scheme of wavelength synthesizing allows the measurement of absolute distances with a high precision that is traceable to the definition of time. The achievable wavelength uncertainty is 1.9×10^{-10} , which allows the absolute heights of gauge blocks to be determined with an overall calibration uncertainty of 15 nm ($k = 1$). These results demonstrate a successful industrial application of an optical frequency synthesis employing a femtosecond laser, a technique that offers many possibilities for performing precision length metrology that is traceable to the well-defined international definition of time.

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1. Introduction

The task of absolute distance measurements intended in this investigation aims to determine distances or lengths straightway up to extensive ranges. Widely used laser interferometers based on homodyne or heterodyne principles are not suitable for this task since they rely on a continuous accumulation of incremental target movements.¹⁻³ Enlarging the equivalent wavelength using a grazing incidence or two-wavelength synthesis is not sufficient to satisfy the usual industrial requirements for ranges and resolutions. Multiwavelength interferometry with continuous wavelength modulation using a tunable diode offers relatively long absolute measurement ranges, but it has not yet reached the precision of relative measurements based on homodyne or heterodyne laser interferometry. Multiwavelength interferometry based on multiple discrete sources of different wavelengths is considered to be the most appropriate measurement technique, but it requires at least three separate monochromatic laser sources whose frequencies must be stabilized in an elaborate manner to the well-defined absorption bands of atoms or molecules. As a result, no convenient tools exist for absolute distance measurements that are commercially available and can be used for general precision engineering purposes.⁴⁻⁶

In recent years, remarkable progress has been made in the field of ultrashort pulse lasers. Along with many applications in various areas of science and engineering, the advent of femtosecond pulse lasers is anticipated to offer opportunities for improving distance measurements, particularly for long-range applications.⁷ In this paper, we describe a new approach of multiwavelength interferometry that uses a femtosecond pulse laser to measure absolute distances with a particular emphasis on enhancing the measurement accuracy for long

ranges. An ultrashort pulse laser provides an optical frequency comb that can be used as a wavelength ruler after it is stabilized to an Rb clock of frequency standard. A temporal scheme for multiwavelength interferometry is demonstrated by tuning an external cavity laser diode consecutively to the preselected light modes of the optical comb. This new approach allows the measurement of absolute distances with a high precision that is traceable to the definition of time, improving the measurement accuracy of absolute distances for industrial uses.

2. Basic principles

2.1 Optical comb of a femtosecond pulse laser

The ultrashort pulse laser adopted for this investigation used a crystal rod made of titanium-doped aluminum oxide (Ti:Al₂O₃, Ti:sapphire) as the gain medium. The Ti:sapphire crystal emitted a wide band spectrum of light spanning a wavelength of 650 to 1100 nm during optical pumping within a resonance cavity, as illustrated in Fig. 1. This extra-wide gain bandwidth led to the generation of an ultrashort pulse train due to the Kerr-lens mode locking that occurred with the self-phase modulation effect of the Ti:sapphire crystal itself.⁸ Pulse durations of less than 1 picosecond could be readily achieved with no external treatment, and these could be further reduced to ~10 femtoseconds with the aid of dispersion compensation prisms.

The ultrashort pulse train of a femtosecond laser appears in the spectral frequency domain as a comb of evenly spaced quasi-monochromatic light modes. This so-called optical comb can be described collectively with two independent parameters, f_r and f_o ,

where f_r represents the mode spacing determined by the repetition rate of the pulses in the time domain and f_o denotes the frequency offset of the optical comb from the absolute zero frequency as a whole, which is caused by the difference between the group and phase velocities of the pulses in the time domain. These two parameters fall in the radio-frequency region of less than 1 GHz. Thus, when these two parameters are stabilized to a well-established radio-frequency standard, such as the Rb clock, the optical frequencies of all the light modes of the comb can be determined to the same precision as the radio-frequency reference, even in an optical frequency regime of a few hundred THz. For example, the optical frequency of the n^{th} mode can be expressed as $f_n = nf_r + f_o$, where n is a very large integer. The current state-of-the-art level of stabilization for f_r and f_o is $\sim 3 \times 10^{-17}$ in the radio frequency domain, which leads to a stabilized f_n at a precision level of 10^{-19} in the optical frequency domain.^{9,10}

2.2 Optical frequency synthesizer

The optical comb of a femtosecond pulse laser consists of a large number of light modes, usually 10^5 – 10^6 , with a single mode ranging from 10 to 100 nW in power. Even though it may be successfully isolated, a single mode is not capable of supplying enough power to an interferometer. For this reason, we adopted an external frequency-tunable diode laser as the working laser source. The frequency of the working laser was tuned by locking to a selected mode of the optical comb. From the many beat signals with different frequencies that were observed when the working laser was mixed with the optical comb, the lowest beat signal f_b was extracted through a low-pass filter with subsequent identification of the corresponding light mode provided by a wavelength meter. This allowed the frequency of the working laser to be expressed as $f_{WL} = nf_r + f_o \pm f_b$, in which the value of n can be precisely determined along with the sign of f_b from reading the wavelength meter in combination with the known values of f_r and f_o locked to a Rb clock. Therefore, by tuning the frequency of the diode laser to produce the preassigned set of n and f_b , a working laser can be synthesized to produce any optical frequency of interest for absolute distance interferometry.

2.3 Multiwavelength interferometry

For a given wavelength λ of the working laser, the absolute distance to be measured can be expressed as $L = (\lambda/2)(m + f)$, where m and f denote an integer ($m = 0, 1, 2, \dots$) and an excess fraction ($1 > f \geq 0$), respectively. The excess fraction can be directly determined by analyzing the resulting interferogram, but the integer m cannot due to the 2π -ambiguity of single-frequency interferometry. For this reason, multiple wavelengths must be provided from the optical synthesizer in sequence so that the absolute distance L can be written in the form of simultaneous equations as

$$L = \frac{\lambda_1}{2}(m_1 + f_1) = \frac{\lambda_2}{2}(m_2 + f_2) = \dots = \frac{\lambda_N}{2}(m_N + f_N) \quad (1)$$

where the subscript N indicates the total number of individual wavelengths in use. Since all of m_i ($i = 1, 2, \dots, N$) must be positive integers, a unique solution of L can be determined in association with a proper estimation of the feasible range of L . In doing so, the required minimum number of wavelengths increases with the extent of the unknown range of L .^{11–13} As a general rule, four equations are sufficient when a good approximation to L is available within an error of less than ± 1.0 mm when $L < 250$ mm.

3. Experiment

3.1 Overall experimental setup

Figure 1 shows a schematic diagram of the overall hardware system configuration used to calibrate the absolute lengths of gauge blocks. The hardware system consisted of three main units: a femtosecond pulse laser source, a tunable external-cavity laser diode (ECLD), and a gauge block interferometer. The femtosecond laser

provided an optical comb for which all the modes were collectively locked to a standard-frequency Rb clock. The ECLD unit produced an independent single-frequency laser beam that was precisely tuned in sequence to a series of selected modes of the optical comb. The ECLD laser was used as the working source for the gauge block interferometer, which was configured to perform the absolute length calibration.

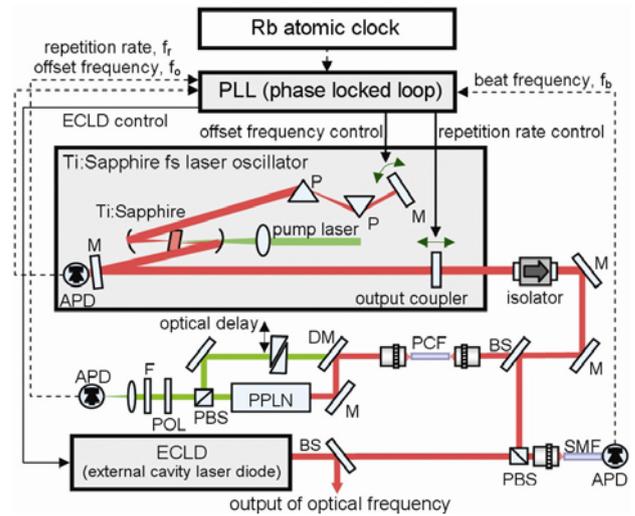


Fig. 1 Construction of an optical frequency synthesizer using the optical comb of a Ti:Sapphire femtosecond laser (M: mirror, P: prism, APD: avalanche photodiode, BS: beam splitter, F: spectral filter, PBS: polarizing beam splitter, DM: dichroic mirror, PCF: photonic crystal fiber, POL: polarizer, SMF: single mode fiber, PPLN: periodically poled lithium niobate)

The femtosecond pulse laser source (Del Mar Photonics, Trestles-50) contained a Ti:sapphire ($\text{Ti:Al}_2\text{O}_3$) crystal and emitted a train of pulses of 35-fs duration with a central wavelength of 780 nm at a repetition rate of 81 MHz. The ultrashort pulse train yielded an optical comb with a spectral width of 24 THz centered at 384 THz and a uniform mode spacing of 81 MHz. To stabilize the optical comb, the pulse repetition rate f_r was locked to a Rb clock signal at 81 MHz by translating the output coupler of the oscillator cavity using the phase locked loop (PLL) control technique. At the same time, the carrier offset frequency f_o was measured with a self-referencing f - $2f$ interferometer and secured to the same Rb clock signal by adjusting the tilt angle of the cavity end mirror. A photonic crystal fiber (Crystal Fibre, NL-PM-750) was used to broaden the spectrum of the optical comb for the f - $2f$ interferometer. This scheme of frequency stabilization, first proposed in 2000⁵, allowed all the comb modes in our hardware setup to be collectively stabilized to a frequency of 1.5×10^{-12} at 10 s. This level of frequency stabilization was sufficient to be used as a wavelength for absolute distance metrology.

The average power of the femtosecond laser was ~ 150 mW. The laser provided about a mere 10 nW to each single mode, which is too weak to be used as the working source for the gauge block interferometer. Therefore, an ECLD (New Focus, TLB-6312) was adopted as the working source; it provided an average power of 12 mW with a line width of less than 300 kHz. The ECLD working laser was continuously tunable within a wavelength range of 765 to 781 nm by varying the external cavity length using a DC motor in tandem with a piezoelectric micro-actuator. The working laser was tuned to the selected mode of the optical comb, first coarsely with a wavelength meter and then finely by locking the beat note of the chosen comb mode to the Rb clock using the PLL technique. The wavelength meter (Angstrom, WS Ultimate 30 Lt) allowed the working laser to be tuned to a resolution of 8 MHz (0.016 pm in wavelength), which is rather coarse but accurate enough to obtain an unambiguous access to the selected optical comb mode. The beat note of the working laser with the selected comb mode was observed for

the fine tuning. Using the PLL technique, the beat note was fixed to a predetermined value by feedback-controlling the current input to the ELCD.

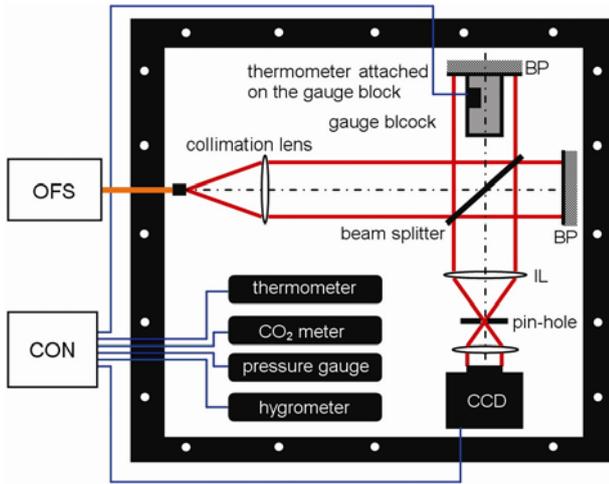


Fig. 2 Optical configuration of the gauge block interferometer (OFS: optical frequency synthesizer, CON: controller, BP: base plate, IL: imaging lens)

A Twyman–Green gauge block interferometer was used, as illustrated in Fig. 2. The working laser was collimated into parallel beams, 50 mm in diameter. The horizontal arm held a flat mirror that reflected a plane wave used as the reference wave. In the vertical arm, the gauge block to be calibrated was wrung onto a flat base plate so that the measurement wave was reflected from the top surface of the gauge block as well as from the base plate. The 2-D interference between the reference and measurement waves was observed using a 640 × 480-pixel CCD camera. Environmental factors such as temperature, pressure, humidity, and CO₂ concentration were measured within the interferometer chamber for a subsequent calibration of the refractive index of air using Edlen's equation. The temperature of the gauge block was monitored to compensate precisely for thermal expansion during the measurement process.

Figure 3 depicts the procedure used to obtain the excess fraction of the gauge block from its sampled interferogram. A reference mirror was intentionally tilted by a small amount to generate sinusoidal carrier fringes at a dominating fringe spatial frequency for the fringe analysis. Three representative lines crossing the carrier fringes were selected: the top (B_a) and bottom (B_b) lines along the base plate, and the middle (G) line along the top surface of the gauge block. The sampled interference intensity data along each selected line were Fourier transformed and their phase values ϕ were subsequently picked up at the peak amplitude corresponding to the spatial carrier frequency. This Fourier-transform method for the fringe analysis allowed the excess fraction f to be determined from

$$f = \frac{1}{360^\circ} \left[\phi_G - \left(\frac{\phi_{Ba} + \phi_{Bb}}{2} \right) \right] \quad (2)$$

Since lines B_a and B_b were separated by the same distance from line G , the phase value for the base plate was determined from the average of ϕ_{Ba} and ϕ_{Bb} without being affected by the non-orthogonal misalignment between the three selected lines and the carrier fringes. The accuracy of this phase measurement requires the amplitude peak to be precisely located in the fringe spatial frequency domain, so the Fourier transformed data were curve-fitted into a Gaussian function, as illustrated in Step 3 of Fig. 3. This process allowed the amplitude peak to be determined with a sub-pixel precision that was less than the original resolution of the Fourier transform.

3.2 Frequency stabilization

Figure 4 shows the results of the frequency stabilization in terms of the Allan deviation. The repetition rate of the optical comb of the femtosecond laser yielded a frequency stability of 1.3×10^{-12} for 10 s of gate time, and reached that of the Rb clock when the gate time increased beyond 1000 s, as shown in Fig. 4(a). The frequency stability of the working laser was 1.9×10^{-10} for 10 s of gate time, as depicted in Fig. 4(b).

3.3 Measurement results and uncertainty evaluation

Four wavelengths of 777, 778, 780, and 781 nm were used to measure a gauge block with a 25-mm nominal length. The excess

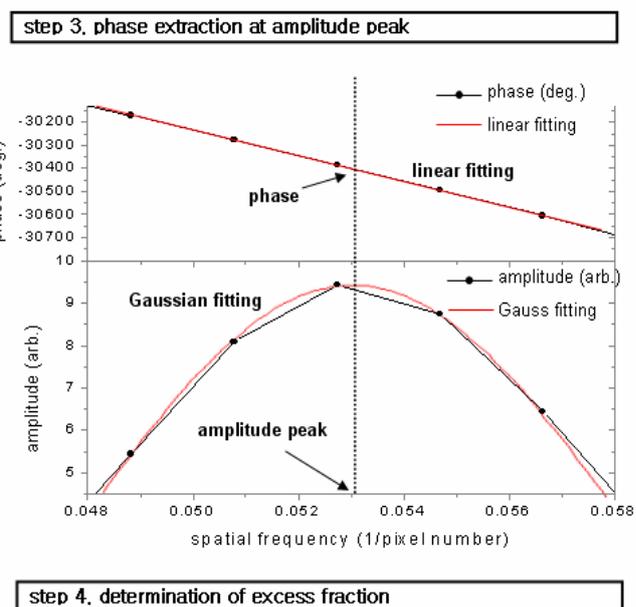
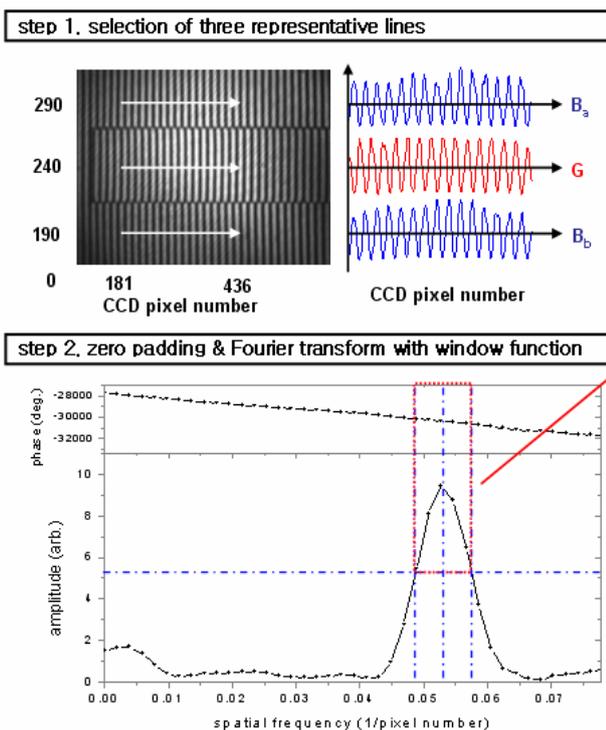
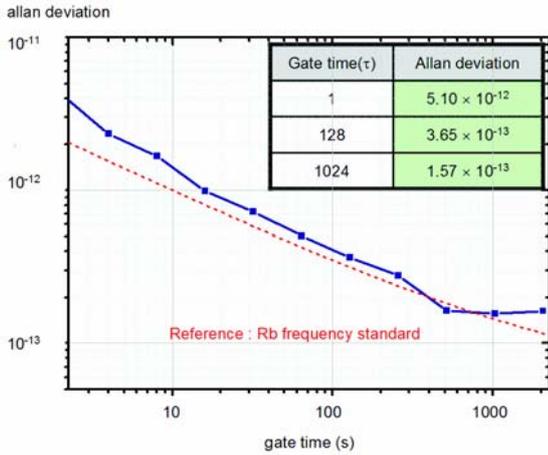


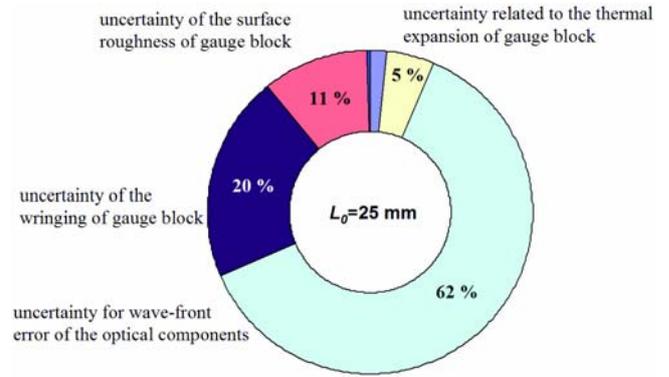
Fig. 3 Fourier transform fringe analysis used to determine the excessive fraction

fraction for each wavelength was measured 20 times repeatedly. Each measurement required 1 s, and they were all averaged to remove the effect of vibrations and thermal fluctuations. The stability of the measured excess fractions was 0.0035. The nominal length of the gauge block was 24.999890 mm; this was the mean of the individual lengths computed from the four wavelengths. The refractive index of air was compensated using the updated Edlen's equation and the actual monitored temperature, pressure, humidity, and CO₂ composition of the air.¹⁴

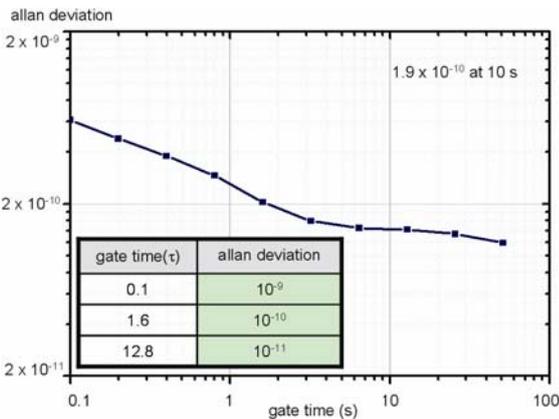
In accordance with conventional and ISO-recommended guidelines¹⁵, an overall uncertainty evaluation was determined for the gauge block calibration. The results are summarized in Table 1, including individual contributions from various sources of error that should be considered. The wavelength uncertainty was $1.9 \times 10^{-10} \cdot L_0$, where L_0 is the nominal length of the gauge block in meters. The wavelength uncertainty yielded an insignificant, practically negligible effect on the overall calibration uncertainty, justifying the use of the optical comb of a femtosecond laser as a wavelength ruler.



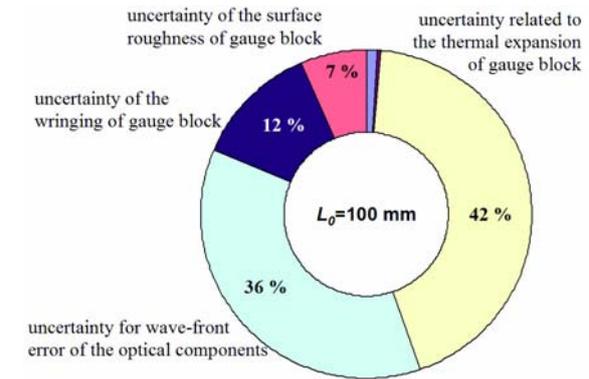
(a) repetition rate of the femtosecond pulse laser



(a) major uncertainty components for $L_0 = 25$ mm



(b) optical frequency synthesizer



(b) major uncertainty components for $L_0 = 100$ mm

Fig. 4 Frequency stabilization test results

Fig. 5 Uncertainty analysis for the length calibration of gauge blocks

Table 1 Uncertainty components for the absolute length measurements based on the optical frequency synthesizer

Sources of uncertainty errors	Value	Value (nm) for $L_0=25$ mm	Value (nm) for $L_0=100$ mm
Uncertainty for the gauge block interferometer	$\sqrt{(2.0 \text{ nm})^2 + (1.9 \times 10^{-10} \cdot L_0)^2}$ nm	2.0	2.0
Uncertainty for the excess fraction part	0.005		
Uncertainty for optical frequency synthesizer	1.9×10^{-10}		
Uncertainty related to the refractive index of air	$1.4 \times 10^{-8} \cdot L_0$ nm	0.4	1.4
Uncertainty for to the thermal expansion of the gauge block	$1.3 \times 10^{-7} \cdot L_0$ nm	3.3	13
Uncertainty for the wave-front error of the optical components of the gauge block interferometer	12 nm	12	12
Uncertainty for the wringing of gauge block	6.9 nm	6.9	6.9
Uncertainty for the surface roughness of gauge block	5.0 nm	5.0	5.0
Uncertainty for the flatness and parallelism of gauge block	0.6 nm	0.6	0.6
Combined standard uncertainty (k=1)	$\sqrt{(15 \text{ nm})^2 + (1.3 \times 10^{-7} \cdot L_0)^2}$	15	20

A major contribution to the uncertainty was due to the fringe-analyzing error induced when determining the excess fraction using Eq. (1). This could be as large as 2.0 nm. The uncertainty for the refractive index of air was $1.4 \times 10^{-8} \cdot L_0$ due to the imprecision of the updated Edlen's equation together with measurement errors pertaining to the temperature, pressure, and composition of the air. The uncertainty in the thermal expansion of the gauge blocks was another major error source that was usually one order of magnitude greater than that of the refractive index of air. The wave-front error of the gauge block interferometer was estimated to reach one-fifteenth of the wavelength, and was the most dominating error source. This was attributable to the imperfection of the optical components, as illustrated in Fig. 5(a). Other practical error sources were caused by the imperfection of wringing the gauge blocks to the base plate and the geometrical error of the gauge blocks themselves, which contributed as much as 6.9 nm to the error.

The uncertainty of the thermal expansion of the gauge block dominated when the multiwavelength interferometer based on the optical frequency synthesizer was used on long blocks with nominal lengths greater than 100 mm, as shown as Fig. 5(b). Because the uncertainty contribution of the optical frequency synthesizer can be ignored for such long-range measurements, the target of this technique can be expanded from millimeter-scale dimensional standards to meter-scale industrial and aerospace applications. Therefore, an optical frequency synthesizer offers various possibilities as a new light-source concept in the field of absolute distance metrology.

4. Conclusions

The concept of using an optical frequency synthesizer to exploit the optical frequency comb of a femtosecond pulse laser was tested by measuring absolute distances. All the light modes of the optical comb were stabilized by locking both the repetition rate and the carrier offset frequency to a commercially available Rb clock of frequency standard. An external cavity diode laser was tuned to provide a sequence of selected wavelengths consecutively, which allowed the performance of a calibration using multiwavelength interferometry. The achieved wavelength uncertainty was 1.9×10^{-10} , which permitted a gauge block calibration with no significant error contributions from the source. These results demonstrate a successful industrial application of an optical frequency synthesis employing a femtosecond laser. Such a technique offers many possibilities for performing precision length metrology with traceability to the well-defined international definition of time.

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